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SPATIAL CORRELATION OF TRANSIONOSPHERIC SIGNAL-TIME-DELAYS. (U)

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SPATIAL CORRELATION OF TRANSIONOSPHERIC
SIGNAL-TIME-DELAYS

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H. Soicher
Communications/ADP Laboratory

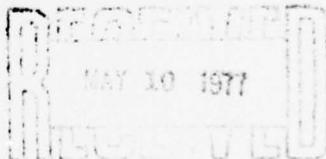
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SPATIAL CORRELATION OF TRANSIONOSPHERIC SIGNAL-TIME-DELAYS

INTRODUCTION

High precision in radar detection, in earth-satellite orbit determination, and in satellite navigation necessitates that the signal data used be corrected for the errors imposed by the ionosphere. Signal-time-delays, or equivalently range errors, are always encountered in transitionospheric measurements because the electromagnetic propagation velocity in the medium is slightly less than the free-space velocity. For frequencies at VHF and above, an excess time delay is inversely proportional to the square of the frequency and is directly proportional to the integrated electron density along the propagation path (i.e., total electron content \equiv TEC). Thus, if TEC is known, or is measured, a perfect correction to ranging can be performed. The TEC may be measured in real time, provided the user has dual-frequency capabilities. Since the ionosphere is a dispersive medium, the relative time delays (or phase differences) between the two frequencies may be used to eliminate the error introduced by the TEC. However, substantial simplification in user equipment could be realized if only one frequency were utilized. Time delays would then be determined by forecasting techniques based on media models. Because of the spatial and temporal variability of the ionospheric electron density, the time-delay errors vary with geographic location, target (or source) altitude, and time. For improved accuracy, the forecasting techniques should be supported by periodic updating of data (preferably in real time) at specified locations. The question arises as to the extent of the geographic area, surrounding a station having real-time TEC-determination capabilities, within which TEC values could be interpolated with acceptable accuracy. In other words, could TEC be determined at location A if a real-time measurement were taken at a different location, B, and what would be the geographic constraints on A and B.

To this end, an investigation designed to determine the correlation between TEC values at Fort Monmouth, N. J. (40.18° N, 74.06° W), and at Richmond, Florida (25.60° N, 80.40° W), was undertaken. Beacon transmissions from the geostationary Applications Technology Satellites ATS-6 ([1] - [3]) were used to determine the TEC at the two stations by means of the Faraday rotation technique.

- [1] H. Soicher, "The ATS-6 Radio Beacon Experiment," *Nature*, vol. 253, Jan. 24, 1975, pp. 252-254.
- [2] K. Davies, R. B. Fritz, and T. B. Gray, "Measurement of columnar electron contents of the ionosphere and plasmasphere," *J. Geophys. Res.*, vol. 81, June 1, 1976, p. 16.
- [3] H. Soicher, "Ionospheric and plasmaspheric effects in satellite navigation systems," *IEEE Trans. Antennas & Propagation*, (in the press).

The Faraday polarization rotation technique has long been used to measure TEC. In the high-frequency and quasi-longitudinal approximations, the two magneto-ionic nodes are nearly circularly polarized, but in opposite senses; thus a plane polarized wave traversing the ionosphere may be regarded as the vector sum of the ordinary and extraordinary components. Since these two components travel at different phase velocities, the plane of polarization rotates continually along the signal's path. The total rotation from the signal source to the observer is related to the total electron content by the expression:

$$a = \frac{k}{f^2} \int_0^S B \cos \theta N ds = \frac{k}{f^2} \int_0^S (B \cos \theta \sec \chi) N dh, \quad (1)$$

where $k = 2.36 \times 10^{-5}$, B is the local magnetic-field flux density in gammas (γ), θ is the angle between the radio wave normal and the magnetic field direction, and χ is the angle between the wave normal and the vertical. Since B decreases inversely with the cube of the geocentric distance, and since the electron density decreases exponentially with altitude above F_2 max (~ 300 km), the integral is heavily weighted near the earth and is therefore considered to provide electron content values for altitudes below ~ 1500 km.

The term $M = B \cos \theta \sec \chi$ in Eq. (1) may be taken out of the integral sign and replaced by its value at a "mean" ionospheric altitude (420 km). Equation (1) then becomes:

$$a = \frac{k}{f^2} \bar{M} \int_{\text{Ionosphere}}^S N dh = \frac{k}{f^2} \bar{M} N_F, \quad (2)$$

where N_F is the ionospheric TEC in el/m^2 measured by the Faraday rotation technique.

The subionospheric points for the two stations [i.e., the geographic locations where the ray paths to the ATS-6 (located at 94° W) intersect a "mean" altitude of 420 km] were 36.5° N, 76.6° W and 23.6° N, 81.6° W, respectively. Thus the "representative" TEC for the two stations was separated by $\sim 13^{\circ}$ in latitude and by 5° in longitude (corresponding to a 20-minute difference in local time).

The daily variations of vertical TEC measured by the Faraday technique for the representative month of February 1975 at Fort Monmouth and at Richmond are shown in Figs. 1 and 2, respectively.

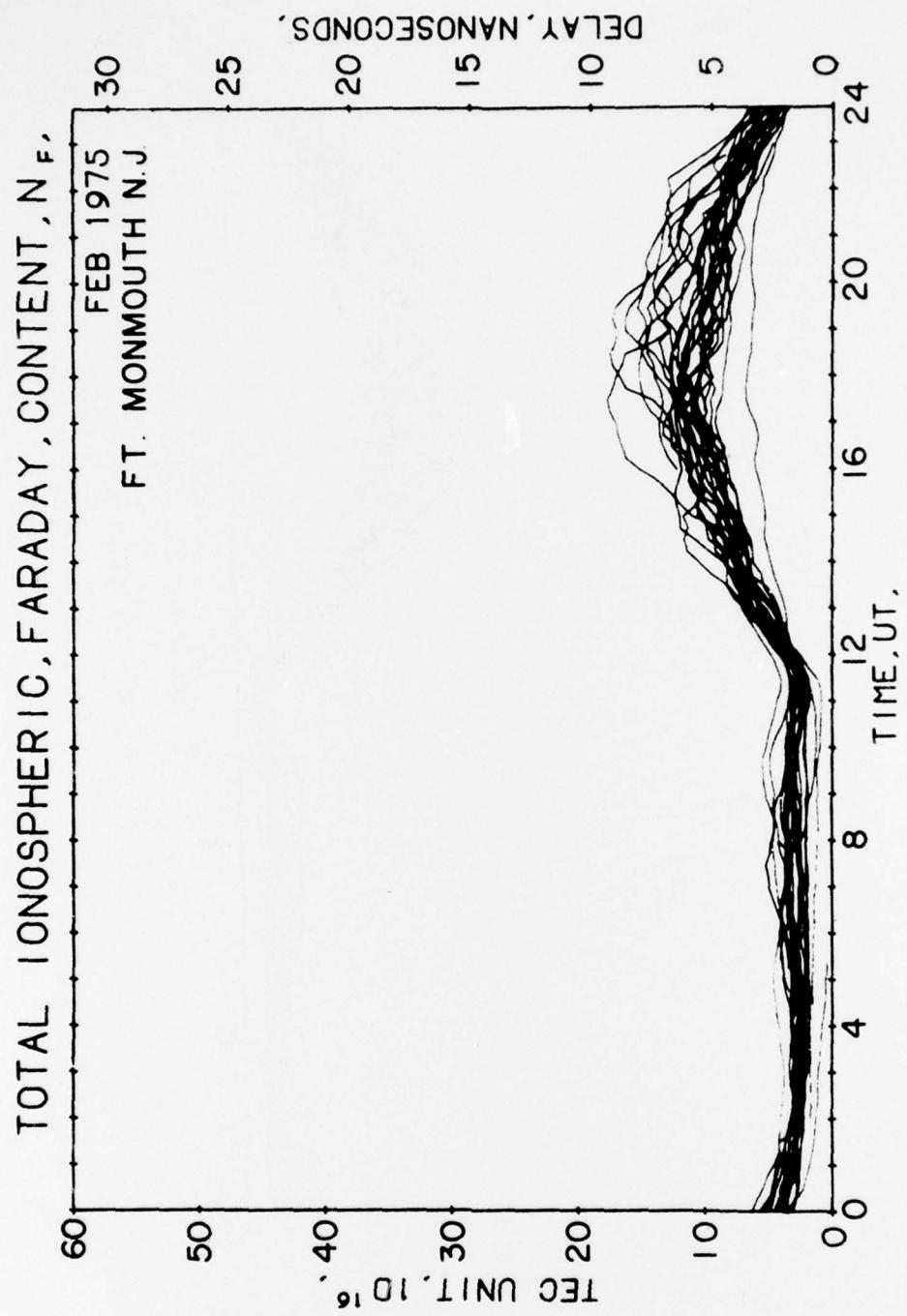


Fig. 1. Total ionospheric (Faraday) electron content (N_F) at Fort Monmouth, N. J., February 1975 (left ordinate: 1 TEC unit = 10^{16} e ℓ /m 2 , right ordinate: time delay normalized to 1.6 GHz).

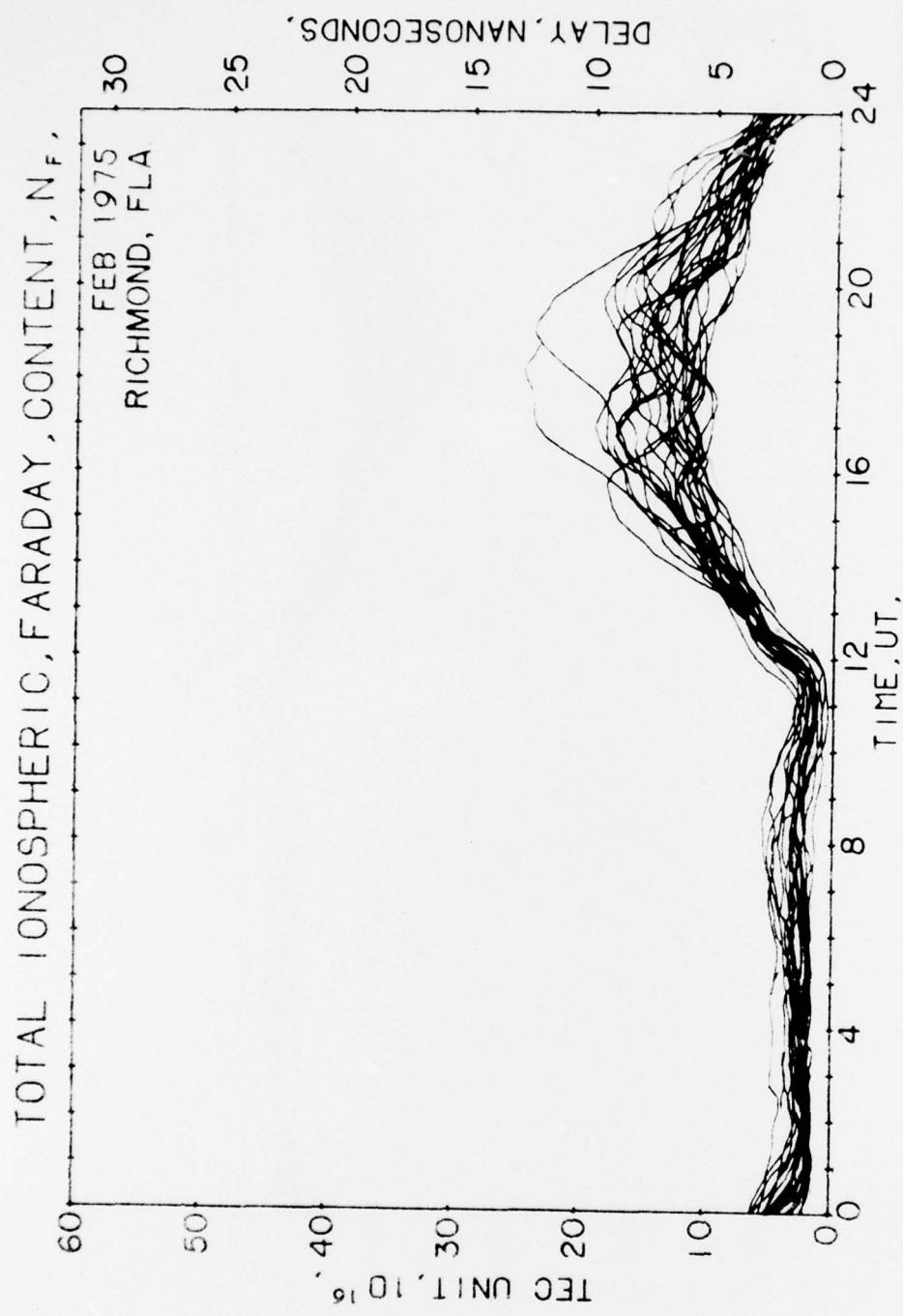


FIG. 2. Total ionospheric ("faraday") electron content (N_F) at Richmond, Florida, February 1975
 (left ordinate; TEC unit = $10^{16} \text{ e}/\text{cm}^2$; right ordinate: time delay normalized to 1.6 GHz).

The equivalent ionospheric signal-delay-times normalized to a frequency of 1.6 GHz (in the satellite navigation frequency band) are also indicated in these figures. The normal diurnal variation of TEC is evident, as is its day-to-day variability. On the average, the Fort Monmouth TEC varied from ~ 2.5 TEC units (1 TEC unit $\equiv 10^{16}$ esu/m 2) prior to dawn, to ~ 12 TEC units in the early afternoon, while the Richmond TEC varied from ~ 2.3 TEC units to ~ 14.2 TEC units at similar times. The day-to-day variability of TEC, as determined from the standard deviation of the daily TEC from the monthly mean, ranged from ~ 0.55 to ~ 2.7 TEC units at Fort Monmouth and from ~ 0.6 to ~ 3.0 TEC units at Richmond. The standard deviation minimized and maximized at approximately the same times as the TEC.

In the next sections, the correlation technique applied to the two sets of TEC data will be described, and the results and conclusions drawn from the correlation analysis will be presented. The analysis was performed for the first five months of 1975 (the satellite drifted away from the U.S. horizons in late May 1975), and, for additional seasonal comparison purposes, for the month of September 1974.

CORRELATION ANALYSIS

The correlation analysis was done in the following manner. The TEC data for Fort Monmouth, N.J., and Richmond, Florida, denoted the two correlated variables. A least squares regression line was fitted to the two data sets with the time parameter eliminated between them. The coefficient of correlation, r , is defined as:

$$r = \pm \sqrt{\frac{\sum (Y_{est} - \bar{Y})^2}{\sum (Y - \bar{Y})^2}}$$

where Y_{est} represents the value of Y (TEC at Fort Monmouth) for a given value of X (TEC at Richmond) estimated from the fitted regression line; \bar{Y} is the mean of Y .

The correlation coefficient for the data was completed in two ways--in *monthly intervals* and in *daily intervals*. For the *former way*, a full-month's TEC values for Richmond were correlated with a full-month's TEC values for Fort Monmouth. The first calculation was performed for the two sets with no shift in time between them (i.e., TEC values for a full month at Fort Monmouth were correlated with TEC values for the same month and same time at Richmond). The next calculation was performed with the monthly Richmond data, relative to the monthly Fort Monmouth

data, shifted forward in time by 15 minutes. The two end data points (i.e., first monthly Fort Monmouth data point and last monthly Richmond data point) no longer had corresponding data points and were therefore eliminated from this calculation. The ensuing calculation was performed with another 15-minute forward shift of the Richmond data with respect to the Fort Monmouth data. After ninety-six such consecutive 15-minute shifts, the correlated data pairs are missing the first day of the month at Fort Monmouth and the last day of the month at Richmond. The total time shift for the complete calculation was ten days at 15-minute intervals. The same analysis was performed with the Richmond, Florida, monthly data set correlated with itself for the month of September 1974. The no-shift calculation yields, of course, a correlation coefficient of +1.

For the latter way, data sets for both locations were correlated on a daily-interval basis. Again, the first calculation was performed for the same time at Fort Monmouth and at Richmond, but this time for daily intervals. For the second and ensuing calculations, the data sets for Richmond were shifted backward and forward up to ten times consecutively in each direction in 15-minute shifts.

In addition, the daily data sets for Fort Monmouth for an entire month were correlated with one specific daily data set for Fort Monmouth (the first day of the month for which a complete data set was available). Finally, daily data sets for an entire month at Richmond, Florida, were correlated with respect to one specific daily data set for that month at Fort Monmouth (again the first day of the month for which a complete data set was available).

THE DATA

A typical variation of the monthly correlation coefficient obtained by shifting the monthly TEC data set for Richmond forward in 15-minute intervals with respect to the Fort Monmouth data set is shown in Fig. 3. The maximum positive correlation coefficient was reached for no shift (i.e., the two data sets were correlated for actual corresponding times). The correlation coefficient decreased monotonically to a minimum after 21 fifteen-minute shifts. The maximum negative correlation was reached after 47 such shifts, followed by another minimum after 72 shifts. The correlation coefficient reached another relative maximum after 95 shifts. Roughly speaking, the correlation coefficient reaches a positive maximum at \sim 24-hour intervals, a negative maximum at \sim 12-hour intervals, and minima at \sim 6-hour and \sim 18-hour intervals. This, of course, is to be expected, owing to the diurnal variation of the data sets.

The monthly correlation analysis for September 1974 and January, February, March, April, and May 1975 is summarized in Fig. 4. Indicated in this figure are the positive maxima of

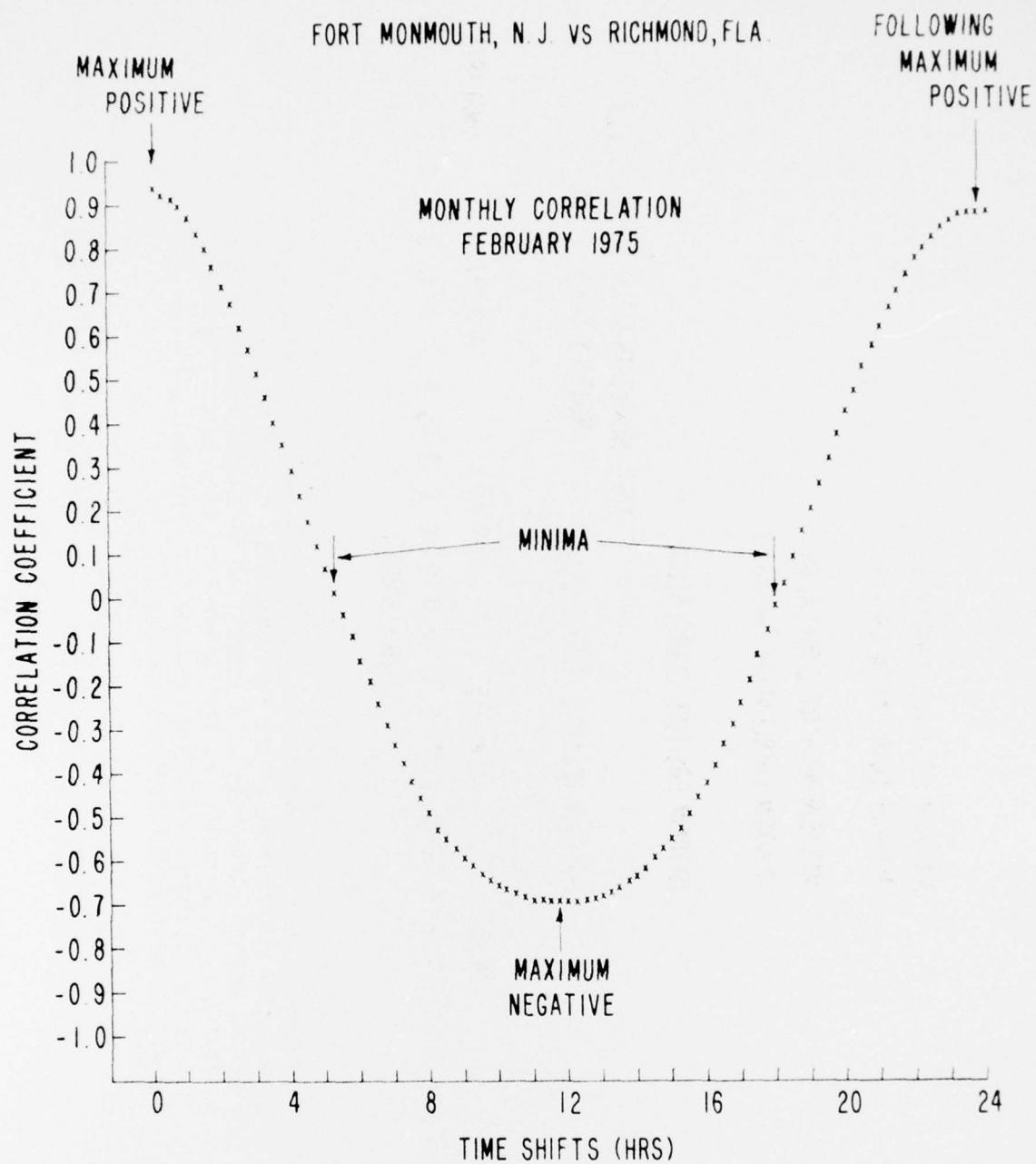


Fig. 3. Variation of the correlation coefficient of the Fort Monmouth, N.J., and Richmond, Florida, monthly data sets with respect to 15-minute time shifts (first data point corresponds to no shift, while last data point corresponds to 24-hour forward shift between data sets).



Fig. 4. Variation of the positive maxima, negative maxima, and minima of the correlation coefficient of the Fort Monmouth, N. J., and Richmond, Florida, monthly data sets with respect to daily time shifts. Also indicated are the corresponding quarter-hour time shifts for which these were attained. Analysis was made for September 1974 and January, February, March, April and May 1975.

the correlation coefficient, the negative maxima of the correlation coefficient, and the corresponding time shifts for which these were attained. These time shifts are plotted as daily shifts, since the correlation coefficients maximize (positively or negatively) at ~ 24 -hour periods. Also indicated are the time shifts that correspond with minimum correlation ($< |0.03|$).

For all indicated months, the positive maximum of the correlation coefficient was highest ($\gtrsim 0.9$) for corresponding data sets (i.e., no shift in time). The positive maximum of the coefficient for succeeding time shifts declined; however, not monotonically. The maximum of the correlation coefficient for January and February, in general, was higher than that for the other months. The range of the coefficient for all reported time shifts (except zero shifts) was between ~ 0.7 and ~ 0.9 . The range of the corresponding time shifts was 96 ± 2 shifts.

In general, for all the indicated months, the negative maximum of the correlation coefficient was not the highest for the fewer number of shifts. The lowest values were observed for January and February. For corresponding time shifts during September 1974, the negative coefficient exceeded the positive coefficient in magnitude, but during January and February, the opposite was true. During March, April and May 1975, the magnitude of the negative and positive maxima of the coefficients were comparable, but their relative values alternated in magnitude. The range of the negative maxima of the coefficient for all reported time shifts was between ~ -0.6 and ~ -0.9 . The range of the corresponding time shifts was 48 ± 2 shifts.

The minima of the correlation coefficient were reached at time shifts which ranged between 21 and 27 and between 71 and 76. For the former, a greater number of shifts was observed during September, April and May than during the other months.

The monthly variation of the correlation coefficient for the Richmond data during September 1974 correlated with itself is shown in Fig. 5. The magnitude of the correlation coefficient for the no-shift condition, of course, is +1.0, since the two data sets are identical. The positive maxima of the correlation coefficient varied between ~ 0.7 and ~ 0.84 , while the time shifts for their attainment ranged at 96 ± 2 shifts. (In general, these positive maxima of the correlation coefficient were higher in value than the corresponding Fort Monmouth-Richmond coefficient for the same month.) The negative maxima of the correlation coefficient varied between -0.68 and -0.78, while the time shifts for their attainment ranged between 43 and 48 shifts. (The negative maxima of the correlation was always lower in value than the corresponding Fort Monmouth-Richmond coefficient for the same month.) Time shifts for the minima of the coefficient varied between 21 and 24 and between 71 and 76 shifts.

The day-by-day (daily) maximum positive correlation coefficients for September 1974 and January, February, March, April

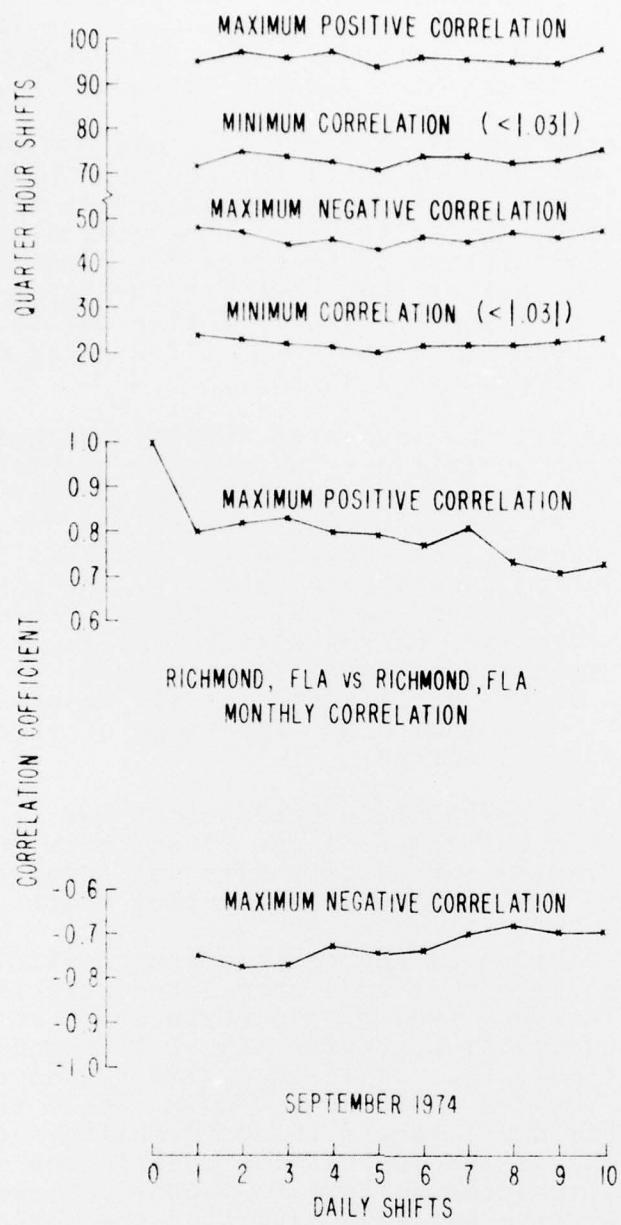


Fig. 5. Variation of the positive maxima, negative maxima, and minima of the correlation coefficient of the Richmond, Florida, data set with itself with respect to daily time shifts between the data sets for September 1974.

and May 1975 are shown in Fig. 6. These were arrived at by comparing the Fort Monmouth and Richmond data sets between 0000 and 2345 UT for each day of those months. The Richmond data was shifted with respect to the Fort Monmouth data at 15-minute intervals in the forward (+) direction and in the backward (-) direction. The number of shifts for which the coefficients are maximized is also shown in the figure, as are the shifts' monthly averages. In addition, the number of data pairs available for the correlation analysis for each day (maximum of 96 data pairs, since the data is available at 15-minute intervals) is also shown in the figure.

In general, the correlation coefficients ranged between ~ 0.9 and ~ 1.0 with relatively few falling below 0.9. The lower values of the coefficients did not necessarily coincide with the sparsity of the available data. On the average, the coefficient was maximum for no shifts in September, for ~ 1 shift in January, and for $> (-1)$ shift for the other months. While most maximum coefficients occurred for ± 1 -hour shifting (± 4 fifteen-minute shifts), shifts of two hours and more were observed occasionally. In general, the maximum coefficients of the daily correlations were higher than the corresponding maxima of the coefficient of the monthly correlations. This was apparently caused by the day-to-day variations throughout the different months which tended to lower the correlation coefficient as the two data sets were shifted with respect to each other.

The maximum correlation coefficients for the daily Fort Monmouth data sets correlated with one specific day for each of the reported months is shown in Fig. 7 (e.g., the daily data sets for September 1974 at Fort Monmouth are correlated individually with the daily data set for September 1, 1974 at Fort Monmouth). Also shown in the figure are the 15-minute time shifts for which the maximum correlations were attained, and the maximum data pairs available for the correlation analysis. The maximum correlation coefficients ranged between ~ 0.85 and ~ 1.0 , with two exceptions. The deviation from the mean was least in March; it was largest in September and April. On the average, the time shifts for maximum correlation were $< +2$ for September, January and April, 0 for March, and > -3 for February and May. Individual shifts, however, reached 10. (The maximum correlation coefficients were comparable in magnitude to those daily correlations between Fort Monmouth and Richmond.)

The maximum correlation coefficients for daily Richmond data sets correlated with one specific daily data set for Fort Monmouth for September 1974 and January through April 1975 are shown in Fig. 8 (e.g., the September 1974 daily Richmond data sets are correlated individually with the September 1, 1974, Fort Monmouth data set). Also shown in the figure are the fifteen-minute time shifts for which the maximum coefficients were attained, and the maximum available data pairs for the correlation analysis. The maximum correlation coefficients ranged between ~ 0.7 and ~ 1.0 for September 1974 and January and April 1975, and between ~ 0.9

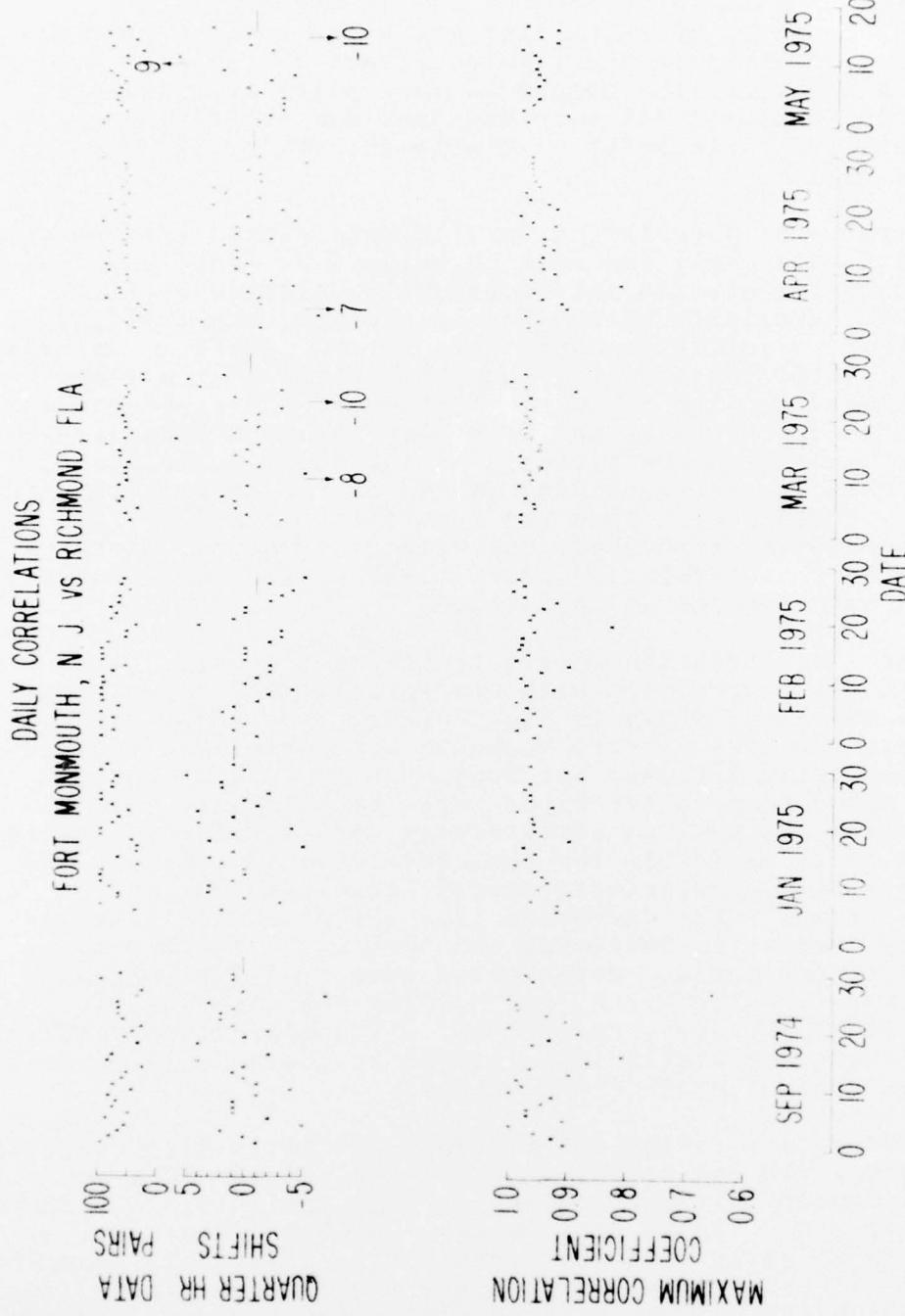


Fig. 6. Variation of the maximum correlation coefficients of the Fort Monmouth, N.J., and Richmond, Florida, daily data sets. Also indicated are the time shifts for which these were attained, their averages (—), and the number of data pairs used in the analysis.

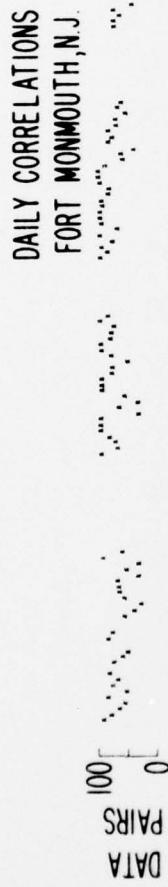


Fig. 7. Variation of the maximum correlation coefficients of the Fort Monmouth daily data sets with respect to one specific daily data set for that location. Also indicated are the time shifts for which these were attained, their averages (—), and the number of data pairs used in the analysis.

DAILY CORRELATIONS
FORT MONMOUTH, N.J. VS RICHMOND, FLA.

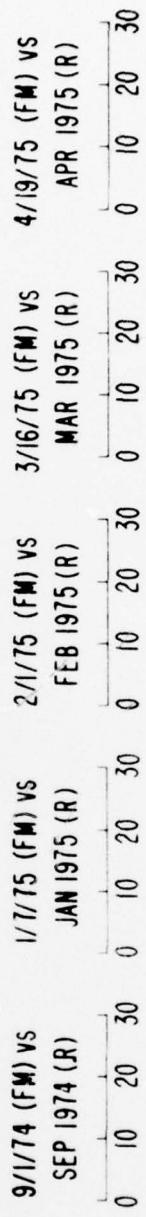


Fig. 8. Variation of the maximum correlation coefficients of the Richmond daily data sets with respect to one specific daily data set at Fort Monmouth. Also indicated are the time shifts for which these were attained, and the number of data pairs used in the analysis.

and ~ 1.0 for February and March, 1975. The scatter of the correlation coefficients was again lowest in March and highest in September and in April. On the average, the time shifts for maximum correlation were < -2 for September, ~ 0 for January and March, $> +1$ for February, and ~ -4 for April. Individual shifts reached ± 10 .

SUMMARY AND CONCLUSIONS

The data reported indicates that TEC, or equivalently, ionospheric signal time-delays, are highly correlatable in two locations, separated by $\sim 13^\circ$ in latitude and $\sim 5^\circ$ in longitude (approximately 2000 km).

The correlation analysis was performed for two localities differing slightly in local time (or equivalently, in longitude) during the quiet phase of the solar cycle. Additional analyses should be performed for two stations with a wider geographic separation in latitude and longitude. Further, it should be done with data taken during periods of increasing solar activity, when ionospheric effects maximize, to ascertain solar cycle effects on the correlatability of TEC.

When the two monthly data sets were compared, a correlation coefficient of ~ 0.9 was obtained for identical times. Time shifts of one data set with respect to the other of the order of ~ 24 hours yield positive maxima of the correlation coefficient which were almost always > 0.7 . Similarly, time shifts of the order of ~ 12 hours yield negative maxima of the correlation coefficient of $\sim (-0.6)$, while time shifts of ~ 6 and ~ 18 hours yield minima of the correlation coefficient.

A seasonal variation in the correlation coefficient was discerned. The positive maxima of the coefficient were lower during the equinoctial periods and higher during the winter, while the negative maxima of the coefficient were more negative during the equinoctial periods than during the winter period. When a monthly data set at one location was compared to itself ($r = +1$ for the no time-shift condition), the positive maxima of the correlation coefficient were always > 0.7 and the negative maxima of the correlation coefficient were always $> (-0.68)$ for time shifts of ~ 24 hours and ~ 12 hours, respectively. Minima of the correlation coefficient were attained for time shifts of ~ 6 and ~ 18 hours. The positive maxima of the correlation coefficient at one location was higher than the corresponding two-location coefficient obtained for the same time period, while the negative of the maxima correlation coefficient was less negative than the corresponding two-location coefficient.

It follows that the continuous TEC data at one of the locales may be used to forecast a continuous data set at the other locale (provided some corresponding data is available at the other locale) so that the two data sets will be correlated with a coefficient of > 0.7 .

When the daily data sets for the two locales were compared, the maximum correlation coefficients were, in general, $\gtrsim 0.9$. The correlation coefficient was > 0.95 during 16 September 1974 when the TEC at both locales was greatly enhanced (by a factor of ~ 2.5 with respect to the monthly average) due to a severe magnetic storm. It follows that TEC values at one locale may be ascertained at any time from a short term (~ 1 day) known current variation of TEC at the other locale. While the time shifts required to attain the maximum coefficients vary from day-to-day and from month-to-month, they ranged on the average between ± 15 minutes from the no-shift condition.

Total electron content values at a locale may be ascertained for a period of one month from one daily TEC data set at the same locale so that the maximum coefficients of correlation are generally > 0.85 . While the time shifts required to attain the maximum coefficients vary from day-to-day and from month-to-month, they range, on the average, between ± 30 minutes from the no-shift condition.

Finally, TEC values at one locale may be ascertained for a period of one month from one daily TEC data set at the other locale, so that the maximum correlation coefficients are generally > 0.7 , and for some months $> \sim 0.9$. While the time shifts required to attain the maximum coefficients vary from day-to-day and from month-to-month, they range, on the average, between ± 30 minutes from the no-shift condition (except for April 1975, when the average time shift was -1 hour).

These specific results for the Fort Monmouth-Richmond locale separation give credence to the lower complexity, one-frequency alternative mentioned in the Introduction. A satellite navigation system using one frequency could provide accurate ranging information by correcting for ionospheric errors. Ionospheric predictions at a specific locale based on continuous TEC real-time measurements at another locale may be made.

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